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GENERALITIES OF PASSIVE VIBRATION DAMPERS ISOLATING VIBRATIONS

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Methods are proposed for investigating complex dynamic systems, whose equilibrium equations are reduced to two coupled inhomogeneous second-order equations with indivisible variables. It is shown that the properties of such mechanical systems can be investigated on the basis of general rules. Relations between the physical parameters of the indicated systems, which substantially simplify cumbersome numerical calculations and lead to simple analytical formulas, have been derived. The methods proposed can be used for investigating the generalities of passive uniaxial vibration dampers.

The problem on isolation of elements of mobile and stationary machines from different vibrations is ambiguous. On the one hand, it is necessary to use a soft suspension in a mobile machine to protect it from the vibrations caused by the unevenness of a road or another base, and, on the other, a rigid suspension should be used to provide a good contact between the machine and the road. Road unevennesses give rise to vibrations with a broadband continuous frequency spectrum, in which there always appear resonance frequencies. A soft suspension having only elastic properties aids in undesirable increasing of the amplitude of these vibrations.

The vibration-isolation system of the engine (power-generating set) of a mobile machine should have higher eigenfrequencies as compared to those of the suspensions of this machine and, at the same time, possess a low dynamic stiffness in the process of small relative displacements of the engine and provide a good damping in the case of resonance.

In mobile machines, metal springs, salient blocks, rubber-metal shock absorbers, and new-generation shock absorbers — hydraulic supports — are used as passive (with invariable physical properties) uniaxial vibration isolators. These isolators are preliminarily designed with the use of simplest elastic and damping elements, the properties of which change linearly depending on the displacements and velocities. A general dynamic system of vibration isolation is presented in Fig. 1. The equations of motion for this system represent two coupled inhomogeneous differential equations with indivisible variables:

$$\ddot{x}_1 = b_{11}\dot{x}_1 + b_{12}\dot{x}_2 + c_{11}x_1 + c_{12}x_2 + f_1 \,, \tag{1}$$

$$\ddot{x}_2 = b_{21}\dot{x}_1 + b_{22}\dot{x}_2 + c_{21}x_1 + c_{22}x_2 + f_2 \,, \tag{2}$$

where $b_{11} = -(b_1 + b_3 + b_5)/m_1$, $b_{12} = b_5/m_1$, $b_{21} = b_5/m_2$, $b_{22} = -(b_2 + b_4 + b_5)/m_2$, $c_{11} = -(c_1 + c_3 + c_5)/m_1$, $c_{12} = c_5/m_1$, $c_{21} = c_5/m_2$, $c_{22} = -(c_2 + c_4 + c_5)/m_2$, $f_1 = F_1/m_1$, $f_2 = F_2/m_2$, and F_2 can be proportional to F_1 with a coefficient α and is opposite to it in direction: $F_2 = -\alpha F_1$.

Such equations are widely used in mechanics. They define the wobbling motion of a ship and its gyroscope, the vibrations of frictionally coupled vibrational systems, the absolute motion of a horizontal pendulum, and the vibrations of objects isolated from vibrations. The above-indicated system of differential equations cannot be integrated in the general form. Solutions can be obtained only in the case where the right sides of these equations are specifically selected and certain conditions are imposed on the coefficients b_{ij} and c_{ij} (i = 1, 2; j = 1, 2) [1]. Present-day computers make it possible to obtain numerical results only for small time intervals and concrete parameters of the problem, which gives no way of deducing the influence of the coefficients of the indicated equations on the total vibrational

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process. The discrepancy between the differential equations comprising system (1)–(2) leads to the appearance of errors and to termination the of calculation in a limited number of steps. The trouble with system (1)–(2) is the connectedness of the variables and their first and second derivatives. Let us separate the derivatives, using the operation of repeated differentiation, which is not contradictory to the Peano existence theorem [1]. Then, if the right sides of system (1)–(2) with harmonic trigonometric functions are differentiable, the following disconnected system of two fourth-order differential equations is obtained for problems on vibration isolation:

$$x_1^{\text{IV}} + \Delta_3 x_1''' + \Delta_2 x_1'' + \Delta_1 x_1' + \Delta_0 x_1 = f_1'' + \Delta_{f'}^2 + \Delta_f^2 , \tag{3}$$

$$x_2^{\text{IV}} + \Delta_3 x_2''' + \Delta_2 x_2'' + \Delta_1 x_2' + \Delta_0 x_2 = f_2'' - \Delta_{f'}^1 - \Delta_f^1 . \tag{4}$$

Here

$$\Delta_3 = \begin{vmatrix} b_{11} & 1 \\ b_{22} & -1 \end{vmatrix} = -(b_{11} + b_{22}) ; \quad \Delta_2 = \begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} + \begin{vmatrix} c_{11} & 1 \\ c_{22} & -1 \end{vmatrix} ; \quad \Delta_1 = \begin{vmatrix} c_{11} & c_{12} \\ b_{21} & b_{22} \end{vmatrix} + \begin{vmatrix} b_{11} & b_{12} \\ c_{21} & c_{22} \end{vmatrix} ;$$

$$\Delta_0 = \begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix} ; \quad \Delta_{f'}^2 = \begin{vmatrix} b_{12} & f_1' \\ b_{22} & f_2' \end{vmatrix} ; \quad \Delta_f^2 = \begin{vmatrix} c_{12} & f_1 \\ c_{22} & f_2 \end{vmatrix} ; \quad \Delta_{f'}^2 = \begin{vmatrix} b_{11} & f_1' \\ b_{21} & f_2' \end{vmatrix} ; \quad \Delta_f^1 = \begin{vmatrix} c_{11} & f_1 \\ c_{21} & f_2 \end{vmatrix} .$$

The characteristic equations of system (3)–(4) are identical because a conservative mechanical system has a set of its own physical parameters that are characteristic of it only. The right sides of Eqs. (3) and (4) are different and characterize the forced vibrations of a nonconservative mechanical system. Natural vibrations differ in the numerical parameters of the initial conditions. The general characteristic equation has the form

$$\lambda^4 + \Delta_3 \lambda^3 + \Delta_2 \lambda^2 + \Delta_1 \lambda + \Delta_0 = 0 . \tag{5}$$

The system of differential equations (3)–(4) will be generalized if one of these equations is written with the indices i and j:

$$x_i^{\text{IV}} + \Delta_3 x_i''' + \Delta_2 x_i'' + \Delta_1 x_i' + \Delta_0 x_i = f_i'' + (-1)^j \Delta_{f'}^j + (-1)^j \Delta_f^j , \quad i = 1, 2 , \quad j = 2, 1 , \quad j \neq i . \tag{6}$$

It is a straightforward matter to analyze the new system of differential equations (6): to find the dynamic transfer functions and frequency transfer functions, the amplitude-frequency characteristics, the resonance frequencies, and the coefficients of dynamics, dynamic stiffness, and dynamic compliance, to determine the stability of vibrations of a mechanism by the Routh, Hurwitz, Nyquist, and Mikhailov conditions, and to select an optimum tuning of a passive vibration isolator. For this purpose, we will use the integral Laplace transform with a complex parameter p at zero initial conditions. The functions $X_1(p)$, $X_2(p)$, $Y_1(p)$, and $Y_2(p)$ are the Laplace representations of the originals $x_1(t)$, $x_2(t)$, $F_1(t)$, and $F_2(t)$. Then the dynamic system (6) will take the form

$$(p^4 + \Delta_3 p^3 + \Delta_2 p^2 + \Delta_1 p + \Delta_0)\, X_i = p^2 Y_i + (-1)^j \Delta_{pY(p)-f(0)}^j + (-1)^j \Delta_{Y(p)}^j - p f_i(0) - f_i'(0) , \tag{7}$$

where i = 1, 2 and $j \neq i$ (if i = 1, j = 2; if i = 2, j = 1). The dynamic transfer function $W_i(p) = X_i(p)/Y_i(p)$ (i = 1, 2) is determined from Eq. (7) [2]:

$$W_i(p) = \frac{p^2 Y_i + (-1)^j \Delta_{pY(p)-f(0)}^j + (-1)^j \Delta_{Y(p)}^j - p f_i(0) - f_i'(0)}{(p^4 + \Delta_3 p^3 + \Delta_2 p^2 + \Delta_1 p + \Delta_0)\, Y_i(p)} . \tag{8}$$

If the proportionality condition is fulfilled for $F_i(t)$ (i = 1, 2), the dynamic transfer functions $W_i(p)$ will be independent of the external driving force $F_1(t)$:

$$\Delta^1_{f'} = \begin{vmatrix} b_{11} & f_1' \\ b_{21} & -\alpha f_1' \end{vmatrix} = f_1' \begin{vmatrix} b_{11} & 1 \\ b_{21} & -\alpha \end{vmatrix}, \quad \Delta^1_f = \begin{vmatrix} c_{11} & f_1 \\ c_{21} & -\alpha f_1 \end{vmatrix} = f_1 \begin{vmatrix} c_{11} & 1 \\ c_{21} & -\alpha \end{vmatrix},$$

$$\Delta^2_{f'} = \begin{vmatrix} b_{12} & f_1' \\ b_{22} & -\alpha f_1' \end{vmatrix} = f_1' \begin{vmatrix} b_{12} & 1 \\ b_{22} & -\alpha \end{vmatrix}, \quad \Delta^2_f = \begin{vmatrix} c_{12} & f_1 \\ c_{22} & -\alpha f_1 \end{vmatrix} = f_1 \begin{vmatrix} c_{12} & 1 \\ c_{22} & -\alpha \end{vmatrix},$$

$$\Delta^j_{pY(p)-f(0)} = -(pY_1(p) - f_1(0))(\alpha b_{1j} + b_{2j}), \quad \Delta^j_{Y(p)} = -Y_1(p)(\alpha c_{1j} + c_{2j}),$$

$$\delta_i = \begin{cases} 1, & \text{at } i = 1, \\ -\alpha, & \text{at } i = 2, \end{cases}$$

$$W_i(p) = \frac{\delta_i p^2 + (-1)^{j+1}(\alpha b_{1j} + b_{2j})p + (-1)^{j+1}(\alpha c_{1j} + c_{2j})}{p^4 + \Delta_3 p^3 + \Delta_2 p^2 + \Delta_1 p + \Delta_0} - \frac{(p + (-1)^{j+1}(\alpha b_{1j} + b_{2j}))f_i(0) + f_i'(0)}{(p^4 + \Delta_3 p^3 + \Delta_2 p^2 + \Delta_1 p + \Delta_0)Y_1(p)}. \tag{9}$$

The first term of the dynamic transfer function $W_i(p)$ is independent of the external force $F_1(t)$ and is an eigencharacteristic of a mechanical system, describing the dynamic properties of this system. The second term is determined by the parameters of the driving force at the initial instant of time in the case where a trigonometric dependence, such as $f_1(t) = a_1 \sin \omega t$, takes place and, in combination with the first term of the first fraction, is cancellable with $Y_1(p)$.

The frequency transfer function is obtained from the dynamic transfer function if the parameter p is replaced by $I\omega$ in $W_i(I\omega)$ ($I = \sqrt{-1}$), the amplitude-frequency characteristic expresses the dependence of the amplitude of harmonic forced vibrations $X_1(I\omega)$ and $X_2(I\omega)$ on the frequency of harmonic excitation ω, and $F_1(t) = A_1 \cos \omega t$, where $f_1(t) = a_1 \cos \omega t$:

$$X_i(I\omega) = a_1 \left[\frac{-\delta_i \omega^2 + (-1)^{j+1} I(\alpha b_{1j} + b_{2j})\omega + (-1)^{j+1}(\alpha c_{1j} + c_{2j})}{\omega^4 - I\Delta_3\omega^3 - \Delta_2\omega^2 + I\Delta_1\omega + \Delta_0} - \right.$$

$$\left. - \frac{(I\omega + (-1)^{j+1}(\alpha b_{1j} + b_{2j}))f_i(0) + f_i'(0)}{(\omega^4 - I\Delta_3\omega^3 - \Delta_2\omega^2 + I\Delta_1\omega + \Delta_0)Y_1(I\omega)} \right]. \tag{10}$$

Amplitude-frequency characteristics are represented in the coordinate planes with the use of the modulus $|X_i(I\omega)|$ (i = 1, 2), which, at $f_1 = 0$ and $f_1'(0) = 0$, has the form

$$|X_i(\omega)| = a_1 \sqrt{\frac{[-\delta_i\omega^2 + (-1)^{j+1}(\alpha c_{1j} + c_{2j})]^2 + (\alpha b_{1j} + b_{2j})^2\omega^2}{[\omega^4 - \Delta_2\omega^2 + \Delta_0]^2 + [\Delta_3\omega^3 - \Delta_1\omega]^2}}. \tag{11}$$

An antiresonance can arise at a frequency providing a minimum of the numerator of dependence (11):

$$\omega^2 = \frac{(-1)^{j+1}(\alpha c_{1j} + c_{2j})}{\delta_i}, \quad \alpha b_{1j} + b_{2j} = 0 \tag{12}$$

or in greater detail:

$$\omega^2_{*1} = \frac{c_2 + c_4 + c_5}{m_2} - \frac{\alpha c_5}{m_1} > 0 \quad \text{when} \quad i = 1, \; j = 2 \quad \text{and} \quad \alpha b_{12} + b_{22} \to 0,$$

$$\omega_{*2}^2 = \frac{c_1 + c_3 + c_5}{m_1} - \frac{c_5}{\alpha m_2} > 0 \quad \text{when} \quad i = 2\,, \quad j = 1 \quad \text{and} \quad \alpha b_{11} + b_{21} \to 0\,.$$

If $\omega_{*1}^2 < 0$, an antiresonance is completely absent for $\left|X_1(\omega)\right|$ because the numerator of the radicand

$$\left|X_i(\omega)\right| = a_1 \sqrt{\frac{[\omega^2 + \omega_{*1}^2]^2 + (\alpha b_{12} + b_{22j})^2\,\omega^2}{[\omega^4 - \Delta_2\omega^2 + \Delta_0]^2 + [\Delta_3\omega^3 - \Delta_1\omega]^2}}$$

is always larger than zero (analogously for ω_{*2}^2).

A resonance arises when the denominator in formula (11) is close to zero: $\omega^4 - \Delta_2\omega^2 + \Delta_0 = 0$, $(\Delta_3\omega^2 - \Delta_1)\,\omega = 0$. If the roots of these expressions are equal, the resonance will be infinite. If these roots are different, the resonance will be limited at frequencies

$$\omega_{1,2}^2 = \frac{\Delta_2}{2} \pm \sqrt{\frac{\Delta_2^2}{4} - \Delta_0}\,, \quad \omega_3 = \sqrt{\frac{\Delta_1}{\Delta_3}}\,, \quad \omega_4 = 0\,. \tag{13}$$

The coefficient of dynamics of displacements determines the ratio between the displacements of a mechanical system in dynamics and the displacements of this system in statics. The displacements $x_1(t)$ and $x_2(t)$ in statics are determined from the system of equations (1)–(2): $x_1(t) = R_1 f_1(t)$, $x_2(t) = R_2 f_1(t)$, where $R_1 = -\dfrac{\alpha c_{12} + c_{22}}{c_{11}c_{22} - c_{12}c_{21}}$; $\hat{X}_1 = |R_1|a_1$; $R_2 = \dfrac{\alpha c_{11} + c_{21}}{c_{11}c_{22} - c_{12}c_{21}}$; $\hat{X}_2 = |R_2|a_1$.

The coefficients of dynamics of displacements will be equal respectively to

$$K_{\text{dyn}}^1 = \frac{|X_1(I\omega)|}{\hat{X}_1}\,, \quad K_{\text{dyn}}^2 = \frac{|X_2(I\omega)|}{\hat{X}_2}\,. \tag{14}$$

The dynamic coefficient of accelerations is equal to the ratio between the modulus of acceleration of the output member, determined with account for the elasticity of the members of a mechanism, and the maximum modulus of acceleration of this member, determined without regard for the elasticity of the members of the mechanism:

$$K_{\text{ac}}^1 = \omega^2 \frac{|X_1(I\omega)|}{a_1}\,, \quad K_{\text{ac}}^2 = \omega^2 \frac{|X_2(I\omega)|}{\alpha a_1}\,. \tag{15}$$

At $\alpha = 0$, the dynamic coefficient K_{ac}^2 is not considered because the acceleration $\ddot{x}_2 = 0$.

The real values of the moduli of the dynamic transfer functions $\left|W_i(I\omega)\right|$ are dynamic compliances of mechanical systems of masses m_1 and m_2; the reciprocal quantities $D_i = 1/\left|W_i(I\omega)\right|$ $(i = 1, 2)$ are dynamic stiffenesses.

The Routh and Hurwitz algebraic stability conditions are formed by the characteristic equation (5) [3]. In this case, the motion of a mechanical object will be stable if the four Hurwitz determinants are positive: $\Delta_0 > 0$, $\Delta_3 > 0$, $\Delta_3\Delta_2 - \Delta_1 > 0$, $\Delta_1\Delta_2\Delta_3 - \Delta_0\Delta_3^2 - \Delta_1^2 > 0$.

The frequency criterion of stability of a Nyquist system is verified by the amplitude-phase characteristics (9) constructed in complex planes at $p = I\omega$. A vibration damper will have stable vibrations if the locuses $W_i(I\omega)$ (9) do not cover the point with coordinates $\{-1, 0{\cdot}I\}$ when ω changes from 0 to ∞.

The Mikhailov criterion is formed on the basis of the known characteristic equation (5) or the denominator of the transfer function $W_i(I\omega)$ (9). The locus $Q(I\omega)$ is outlined on a complex plane. By the shape of this locus, the possible stability

$$Q(I\omega) = \omega^4 - \Delta_2\omega^2 + \Delta_0 - I(\Delta_3\omega^3 - \Delta_1\omega) \tag{16}$$

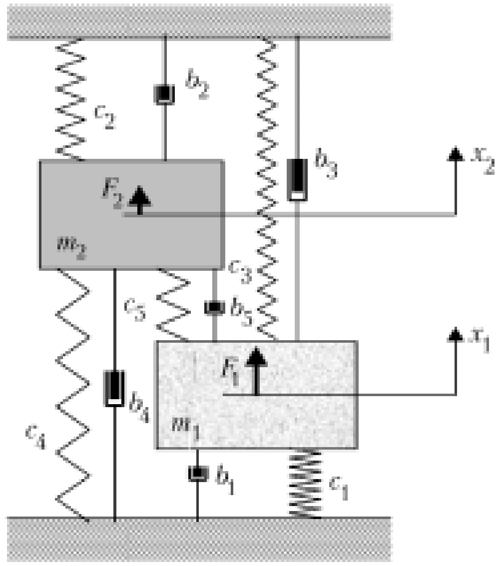

Fig. 1. General dynamic system of vibration isolation.

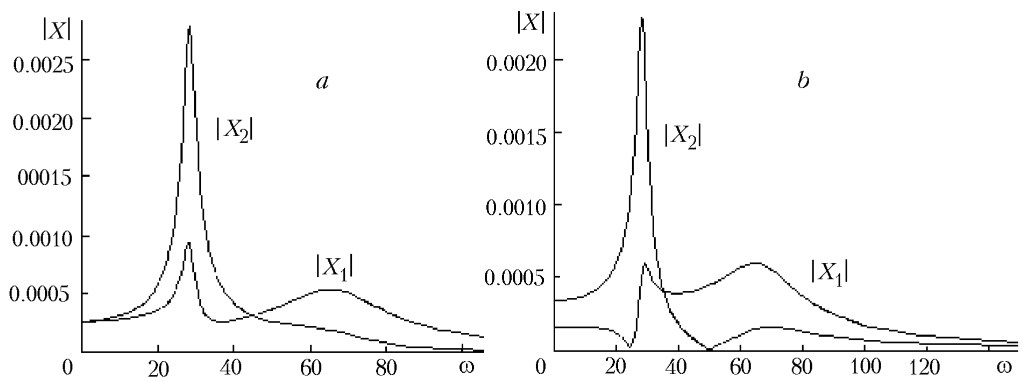

Fig. 2. Amplitude-frequency characteristics of a dynamic spring vibration damper with a viscous friction (a_1 = 1, b_{11} = −20, $b_{12} = -b_{11}$, b_{21} = 10, $b_{22} = -b_{21}$, c_{11} = −4500, c_{12} = 800, c_{21} = 1000, $c_{22} = -c_{21}$): a) $\alpha = 0$, ω_{*1} = 31.6 rad/sec, $\omega_{*2} \to \infty$; b) $\alpha = 0.5$, ω_{*1} = 24.5 rad/sec, ω_{*2} = 50.0 rad/sec.

of the vibration damper designed and the dependence between its parameters, providing this stability, are determined.

The vibration-isolation system shown in Fig. 1 is easily transformed into any other system [4, 5] by removing unnecessary components. If the base is rigid, $m_2 \to \infty$, $c_2 = c_4 = 0$, and $b_2 = b_4 = 0$. If the end is free, $c_2 = 0$ and $b_2 = 0$. If the masses m_1 and m_2 are rigidly connected, $M = m_1 + m_2$, $c_5 = 0$, $b_5 = 0$, and $x(t) = x_1(t) + x_2(t)$. For example, the equation of motion of a dynamic spring damper with a viscous friction has the form of (1) and (2) with coefficients $b_{11} = -b_d/m$, $b_{12} = b_d/m$, $b_{21} = b_d/m_d$, $b_{22} = -b_d/m_d$, $c_{11} = -(c + c_d)/m$, $c_{12} = c_d/m$, $c_{21} = c_d/m_d$, $c_{22} = -c_d/m_d$, $f_1 = F(t)/m$, $f_2 = 0$, and $\alpha = 0$. The dynamic transfer functions are determined from the general equation (9) at $f_1(0) = 0$, $f_1'(0) = 0$, $\delta_1 = 1$, and $\delta_2 = -\alpha$:

$$W_1(p) = \frac{m_d m p^2 + b_d p + c_d}{m m_d p^4 + (m b_d + m_d b_d)\, p^3 + (m_d (c_d + c) + c_d m)\, p^2 + c b_d p + c_d c},$$

$$W_2(p) = \frac{b_d p + c_d}{(mp^2 + b_d p + c + c_d)(m_d p^2 + b_d p + c_d) - (b_d p + c_d)^2}.$$

These expressions completely correspond to the expressions obtained in [2] from the system of two second-order differential equations with indivisible variables with the use of the integral Laplace transform and the general method of solving a system of linear algebraic equations.

According to formula (12), an antiresonance arises at a frequency $\omega_*^2 = c_d/m_d$ of vibrations that are due to the elastic coupling between the mass of a vibration damper and the friction (12). In this case, the amplitude of vibrations (11) is the smallest:

$$\left|X_1(\omega)\right| = \frac{a_1 b_d \omega_*}{\sqrt{[m_d m \omega^4 - (m_d (c_d + c) + m c_d)\,\omega^2 + c_d c]^2 + b_d^2 \omega^2\,[(m_d + m)\,\omega^2 - c]^2}}\,.$$

Figure 2 presents the amplitude-frequency characteristics for arbitrary parameters. According to (12), at $\left|X_1(\omega)\right|$ and $\alpha = 0$ an antiresonance arises at a frequency $\omega_{*1} = \sqrt{|c_{22}|}$, $\omega_{*2} \to \infty$; at $\left|X_2(\omega)\right|$ and $\alpha = 0.5$ an antiresonance arises at a frequency $\omega_{*2} = \sqrt{|c_{11}| - c_{21}/\alpha}$, $\omega_{*1} = \sqrt{|c_{22}| - \alpha c_{12}}$. The resonances are independent of the coefficient α, are determined by formulas (13), and are equal in both variants: $\omega_1 = 28.0$ rad/sec, $\omega_2 = 68.7$ rad/sec, $\omega_3 = 35.1$ rad/sec ($\omega_4 = 0$ rad/sec).

The method proposed is simpler as compared with other methods of investigating complex dynamic systems; it allows one to omit cumbersome mathematical calculations, determine the regularities of the influence of physical parameters on the dynamic properties of a mechanical system, is not contradictory to the existence theorem and the Peano uniqueness theorem, and allows one to determine the common features of passive vibration-isolation dampers. There is another method of analysis of chain dynamic systems or the total mechanical resistance — impedance, which allows one to investigate a vibrational system as a whole by imposing certain restrictions on its dynamic and kinetic parameters, leading to the violation of Newtonian laws.

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NOTATION

A_1, amplitude of vibrations of an external force, H; a_1, amplitude of vibrations of an external force related to the mass m_1, N/kg; c_i, coefficient of elasticity of springs, kg·sec^{-2}; b_i, coefficient of damping elements, kg·sec^{-1}; F_1, external forces, N; m_1, m_2, masses of elements of a vibration isolator, kg; α, δ_j, dimensionless parameters; ω, frequency of vibrations of the external force F_1, rad·sec^{-1}. Subscripts: $i = \overline{1, 5}$, for the elastic and damping coefficients; $j = 1, 2$, numbers of masses; *1 and *2, antiresonances for displacements x_1 and x_2 respectively; d, vibration damper; dyn, dynamics; ac, acceleration.

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